AP Calculus Summer Review Packet

Name: ______________________________ **Date began:** ________ **Completed:** ________

*****A Formula Sheet has been stapled to the back for your convenience!*****

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Complex Fractions

When simplifying complex fractions, work towards one fraction in the numerator (you'll need to use common denominators for this) and one fraction in the denominator (using common denominators). Once you have fraction divided by fraction, you can multiply by the reciprocal. You'll multiply straight across, but should be able to cancel factors common to both the numerator and the denominator.

Example:

$$\frac{-7-\frac{6}{x+1}}{\frac{5}{x+1}} = \frac{\frac{-7(x+1)}{x+1}-\frac{6}{x+1}}{\frac{5}{x+1}} = \frac{\frac{-7x-7-6}{x+1}}{\frac{5}{x+1}} = \frac{\frac{-7x-13}{x+1}}{\frac{5}{x+1}} = \frac{-7x-13}{x+1} \bullet \frac{x+1}{5} = \frac{-7x-13}{5}$$

$$\frac{\frac{-2}{x}+\frac{3x}{x-4}}{5-\frac{1}{x-4}} = \frac{\frac{-2(x-4)}{x(x-4)}+\frac{3x(x)}{(x-4)(x)}}{\frac{5(x-4)}{(x-4)}-\frac{1}{x-4}} = \frac{\frac{-2x+8}{x(x-4)}+\frac{3x^2}{(x-4)(x)}}{\frac{5x-20}{x-4}-\frac{1}{x-4}} = \frac{\frac{3x^2-2x+8}{x(x-4)}}{\frac{5x-21}{x-4}} = \frac{3x^2-2x+8}{x(x-4)} \bullet \frac{x-4}{5x-21} = \frac{3x^2-2x+8}{x(5x-21)}$$

Simplify each of the following.

1. $\dfrac{\frac{25}{a}-a}{5+a}$

2. $\dfrac{2-\frac{4}{x+2}}{5+\frac{10}{x+2}}$

3. $\dfrac{1-\frac{2x}{3x-4}}{x+\frac{32}{3x-4}}$

Functions

To evaluate a function for a given value, simply plug the value into the function for x.

Recall: $(f \circ g)(x) = f(g(x))$ *OR* $f[g(x)]$ read "***f*** of ***g*** of ***x***" Means to plug the inside function (in this case g(x)) in for x in the outside function (in this case, f(x)).

Example: Given $f(x) = 2x^2+1$ *and* $g(x) = x-4$ find *f(g(x))*.

$$\begin{aligned} f(g(x)) &= f(x-4) \\ &= 2(x-4)^2+1 \\ &= 2(x^2-8x+16)+1 \\ &= 2x^2-16x+32+1 \\ f(g(x)) &= 2x^2-16x+33 \end{aligned}$$

Let $f(x)=2x+1$ *and* $g(x)=2x^2-1$**. Find each.**

4. $f(t+1)=$ __________

5. $f\left[g(-2)\right]=$ __________

Let $f(x)=\sin x$ **Find each exactly.**

6. $f\left(\frac{\pi}{2}\right)=$ __________

7. $f\left(\frac{2\pi}{3}\right)=$ __________

Let $f(x)=x^2,\ g(x)=2x+5,$ *and* $h(x)=x^2-1$**. Find each.**

8. $h\left[f(-2)\right]=$ _______

9. $f\left[g(x-1)\right]=$ _______

Find $\dfrac{f(x+h)-f(x)}{h}$ **for the given function *f*.**

10. $f(x)=9x+3$

11. $f(x)=5-2x$

Intercepts and Points of Intersection

To find the x-intercepts, let y = 0 in your equation and solve.
To find the y-intercepts, let x = 0 in your equation and solve.

Example: $y=x^2-2x-3$

$x-\text{int. } (Let\ y=0)$

$0=x^2-2x-3$

$0=(x-3)(x+1)$

$x=-1\ or\ x=3$

$x-\text{intercepts } (-1,0)\ and\ (3,0)$

$y-\text{int. } (Let\ x=0)$

$y=0^2-2(0)-3$

$y=-3$

$y-\text{intercept } (0,-3)$

Find the x and y intercepts for each.

12. $y=2x-5$

13. $y=x^2+x-2$

Use substitution or elimination method to solve the system of equations.
Example:

$$x^2 + y - 16x + 39 = 0$$
$$x^2 - y^2 - 9 = 0$$

<u>Elimination Method</u>

Add the two equations together to eliminate the y-terms

$2x^2 - 16x + 30 = 0$

$x^2 - 8x + 15 = 0$

$(x-3)(x-5) = 0$

$x = 3 \textit{ and } x = 5$

Plug $x = 3$ *and* $x = 5$ into one of the original equations

$3^2 - y^2 - 9 = 0$ $\qquad$ $5^2 - y^2 - 9 = 0$

$-y^2 = 0$ $\qquad$ $16 = y^2$

$y = 0$ $\qquad$ $y = \pm 4$

Points of Intersection: $(3,0)$, $(5,4)$, *and* $(5,-4)$

<u>Substitution Method</u>

Solve one equation for one variable.

$y^2 = -x^2 + 16x - 39$ (1st equation solved for y)

$x^2 - (-x^2 + 16x - 39) - 9 = 0$ Plug what y^2 is equal to into second equation.

$2x^2 - 16x + 30 = 0$ (*The rest is the same as*

$x^2 - 8x + 15 = 0$ *previous example*)

$(x-3)(x-5) = 0$

$x = 3 \textit{ or } x = 5$

Find the point(s) of intersection of the graphs for the given equations.

14. $x + y = 8$
$4x - y = 7$

15. $x^2 + y = 6$
$x + y = 4$

<u>Interval Notation</u>

16. Complete the table with the appropriate notation or graph.

Solution	Interval Notation	Graph
$-2 < x \le 4$		
	$[-1, 7)$	
		

Solve the inequality. State your answer in BOTH interval notation and graphically.

17. $2x - 1 \ge 0$

Domain and Range

Find the domain and range of each function. Write your answer in INTERVAL notation.

18. $f(x) = x^2 - 5$

19. $f(x) = -\sqrt{x+3}$

20. $f(x) = 3\sin x$

21. $f(x) = \dfrac{2}{x-1}$

Inverses

To find the inverse of a function, simply switch the x and the y and solve for the new "y" value.

Example:

$f(x) = \sqrt[3]{x+1}$ Rewrite f(x) as y

$y = \sqrt[3]{x+1}$ Switch x and y

$x = \sqrt[3]{y+1}$ Solve for your new y

$(x)^3 = \left(\sqrt[3]{y+1}\right)^3$ Cube both sides

$x^3 = y + 1$ Simplify

$y = x^3 - 1$ Solve for y

$f^{-1}(x) = x^3 - 1$ Rewrite in inverse notation

Find the inverse for each function.

22. $f(x) = 2x + 1$

23. $f(x) = \dfrac{x^2}{3}$

Also, recall that to PROVE one function is an inverse of another function, you need to show that: $f(g(x)) = g(f(x)) = x$

Example:

If: $f(x) = \dfrac{x-9}{4}$ *and* $g(x) = 4x + 9$ **show *f(x) and g(x)* are inverses of each other.**

$$f(g(x)) = 4\left(\frac{x-9}{4}\right) + 9$$

$$= x - 9 + 9$$

$$= x$$

$$g(f(x)) = \frac{(4x+9) - 9}{4}$$

$$= \frac{4x + 9 - 9}{4}$$

$$= \frac{4x}{4}$$

$$= x$$

$f(g(x)) = g(f(x)) = x$ therefore they are inverses of each other.

Prove *f and g* are inverses of each other.

24. $f(x)=\frac{x^3}{2}$ $g(x)=\sqrt[3]{2x}$

25. $f(x)=9-x^2,\ x\geq 0$ $g(x)=\sqrt{9-x}$

Equation of a line	
Slope intercept form: $y=mx+b$	**Vertical line:** x = c (slope is undefined)
Point-slope form: $y-y_1=m(x-x_1)$	**Horizontal line:** y = c (slope is 0)

26. Use slope-intercept form to find the equation of the line having a slope of 3 and a y-intercept of 5.

27. Determine the equation of a line passing through the point (5, -3) with an undefined slope.

28. Determine the equation of a line passing through the point (-4, 2) with a slope of 0.

29. Find the equation of a line passing through the point (2, 8) and parallel to the line $y=\frac{5}{6}x-1$.

30. Find the equation of a line perpendicular to the y- axis passing through the point (4, 7).

Radian and Degree Measure

Use $\frac{180^\circ}{\pi\, radians}$ to get rid of radians and convert to degrees.	Use $\frac{\pi\, radians}{180^\circ}$ to get rid of degrees and convert to radians.

31. Convert to degrees: a. $\frac{5\pi}{6}$ b. $\frac{4\pi}{5}$ c. 2.63 radians

32. Convert to radians: a. 45° b. -17° c. 237°

Angles in Standard Position

33. Sketch the angle in standard position.

a. $\frac{11\pi}{6}$ b. 230° c. $-\frac{5\pi}{3}$ d. 1.8 radians

Reference Triangles

34. Sketch the angle in standard position. Draw the reference triangle and label the sides, if possible.

a. $\frac{2}{3}\pi$

b. 225°

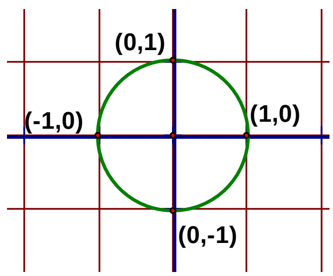
c. $-\frac{\pi}{4}$

d. 30°

Unit Circle

You can determine the sine or cosine of a quadrantal angle by using the unit circle. The x-coordinate of the circle is the cosine and the y-coordinate is the sine of the angle.

Example: $\sin 90^{\circ} = 1$ $\cos\frac{\pi}{2} = 0$

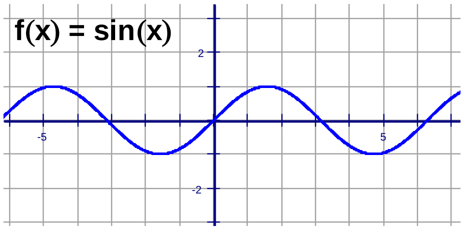


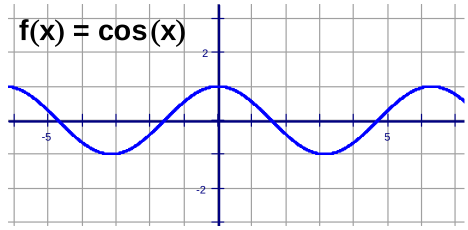
35. *a.*) $\sin 180^{\circ}$ *b.*) $\cos 270^{\circ}$

c.) $\sin(-90^{\circ})$ *d.*) $\sin \pi$

e.) $\cos 360^{\circ}$ *f.*) $\cos(-\pi)$

Graphing Trig Functions





y = sin x and y = cos x have a period of 2π and an amplitude of 1. Use the parent graphs above to help you sketch a graph of the functions below. For $f(x) = a\sin(bx+c)+d$, $|a|$ = amplitude, $\frac{2\pi}{b}$ = period, $\frac{-c}{b}$ = phase shift, and d = vertical shift.

Graph two complete periods of the function.

36. $f(x) = 5\sin x$

37. $f(x) = -\cos\left(x - \frac{\pi}{4}\right)$

Trigonometric Equations:

Solve each of the equations for $0 \le x < 2\pi$. Isolate the variable, sketch a reference triangle, find all the solutions within the given domain, $0 \le x < 2\pi$. Remember to double the domain when solving for a double angle. Use trig identities, if needed, to rewrite the trig functions. (See formula sheet at the end of the packet.)

38. $\sin^2 x = \frac{1}{2}$

39. $\sin 2x = -\frac{\sqrt{3}}{2}$

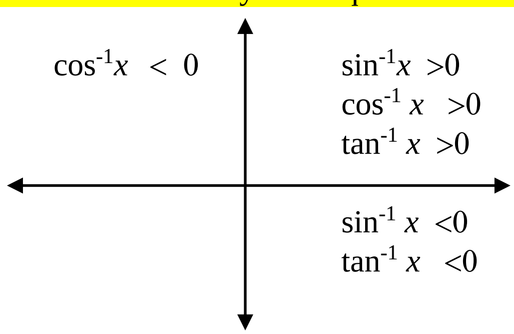
40. $2\cos^2 x - 1 - \cos x = 0$

Inverse Trigonometric Functions:

Recall: Inverse Trig Functions can be written in one of ways:

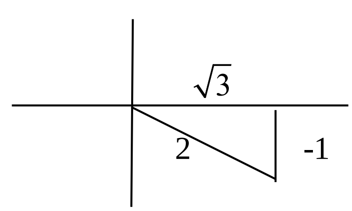
$\arcsin(x)$ $\quad\quad$ $\sin^{-1}(x)$

Inverse trig functions are defined only in the quadrants as indicated below due to their restricted domains.



Example:
Express the value of "y" in radians.

$y = \arctan\frac{-1}{\sqrt{3}}$ Draw a reference triangle. →



NOTE: opposite & adjacent side lengths were labeled to create the tangent above, then missing hypotenuse was found using Pythagorean Theorem.

This means the reference angle is 30° or $\frac{\pi}{6}$. BUT, y = – $\frac{\pi}{6}$ falls in the interval from $\frac{-\pi}{2} < y < \frac{\pi}{2}$

Answer: y = – $\frac{\pi}{6}$

For each of the following, express the value for "y" in radians.

41. $y = \arcsin \frac{-\sqrt{3}}{2}$

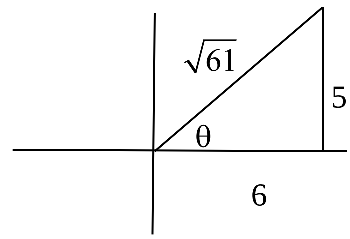
42. $y = \arctan(-1)$

Example: Find the value without a calculator.

$\cos\left(\arctan \frac{5}{6}\right)$

Draw the reference triangle in the correct quadrant first.
Find the missing side using Pythagorean Thm.
Find the ratio of the cosine of the reference triangle.

$\cos\theta = \frac{6}{\sqrt{61}}$



For each of the following give the value without a calculator.

43. $\tan\left(\arccos \frac{2}{3}\right)$

44. $\sin\left(\arctan \frac{12}{5}\right)$

45. $\sin\left(\sin^{-1} \frac{7}{8}\right)$

<u>**Vertical Asymptotes**</u>

Determine the vertical asymptotes for the function. Set the denominator equal to zero to find the x-value for which the function is undefined. That will be the vertical asymptote.

46. $f(x) = \frac{x^2}{x^2 - 4}$

47. $f(x) = \frac{2 + x}{x^2(1 - x)}$

<u>**Horizontal Asymptotes**</u>

Case I. "Top Heavy" There is no horizontal asymptote. (Slant asymptote, using long division, if numerator is exactly one degree larger than denominator)
Case II. "Same Degree" The asymptote is the ratio of the lead coefficients.
Case III. "Bottom Heavy" The asymptote is always y = 0.

Determine all Horizontal Asymptotes.

48. $f(x) = \frac{x^2 - 2x + 1}{x^3 + x - 7}$

49. $f(x) = \frac{5x^3 - 2x^2 + 8}{4x - 3x^3 + 5}$

50. $f(x) = \frac{4x^5}{x^2 - 7}$

Formulas to know and use

TRIG:

Reciprocal Identities: $\csc x = \dfrac{1}{\sin x}$ $\sec x = \dfrac{1}{\cos x}$ $\cot x = \dfrac{1}{\tan x}$

Quotient Identities: $\tan x = \dfrac{\sin x}{\cos x}$ $\cot x = \dfrac{\cos x}{\sin x}$

Pythagorean Identities: $\sin^2 x + \cos^2 x = 1$ $\tan^2 x + 1 = \sec^2 x$ $1 + \cot^2 x = \csc^2 x$

Double Angle Identities: $\sin 2x = 2\sin x \cos x$

$\tan 2x = \dfrac{2\tan x}{1 - \tan^2 x}$

$$\cos 2x = \cos^2 x - \sin^2 x$$
$$= 1 - 2\sin^2 x$$
$$= 2\cos^2 x - 1$$

LOGS:

Logarithms: $y = \log_a x$ is equivalent to $x = a^y$

Product property: $\log_b mn = \log_b m + \log_b n$

Quotient property: $\log_b \dfrac{m}{n} = \log_b m - \log_b n$

Power property: $\log_b m^p = p \log_b m$

Property of equality: If $\log_b m = \log_b n$, then m = n

Change of base formula: $\log_a n = \dfrac{\log_b n}{\log_b a}$

LINES:

Slope-intercept form: $y = mx + b$

Point-slope form: $y - y_1 = m(x - x_1)$

Standard form: Ax + By + C = 0